2015

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

[Mathematical Methods]

(*Marks* : 30)

1. Answer the following questions : 1×4=4

(a) For the complex number $z = 3 - 4i$, find z^4, given that

$$\tan^{-1}\frac{4}{3} = 53 \cdot 13^\circ$$

(b) What does the equation $|z - i| = 2$ represent?

(c) Plot the number $e^{(1-\pi/6^{i})}$.

(d) Find the principal value of i^i.

2. *(a)* Solve the equation $z^4 + 16 = 0$ and plot the values of z. 2

(b) Prove : $\sin^2 z + \cos^2 z = 1$. 2

3. *(a)* Check the analyticity of the function $f(z) = \ln z$ and hence find its derivative. 4

(b) Find the principal value of $(2+i)^{1-i}$. 4

Or

Using Cauchy's integral formula, evaluate the integral

$$\int \frac{z-1}{z^2+1}\, dz$$

around the contours—

(i) $|z-i| = 1$

(ii) $|z| = 2$ 2+2=4

4. *(a)* State and prove Cauchy's integral theorem. 4

(b) Define the following with diagram : 3

(i) Simply connected region

(ii) Multiply connected region

(iii) Equivalent contour

Or

(a) State and prove Taylor's theorem. 4

(b) Find Taylor series expansion about the origin for $\sin \pi z$. 3

5. (a) Define pole and residue. 1

(b) If a function $f(z)$ has an m th order pole at $z = a$, then show that the residue at that singular point is

$$a_{-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left\{ (z-a)^m f(z) \right\}_{z=a}$$

and hence find the residue of

$$f(z) = \frac{e^z}{(z-i)^2}$$

at its pole. 4+2=6

Or

Evaluate the integrals : 3+4=7

(i) $\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2}$

(ii) $\int_0^{2\pi} \frac{\sin\theta \, d\theta}{1+\cos\theta}$

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions : 1×4=4

(a) What is areal velocity of a particle?

(b) The equation of constraint for a particle moving on or out of the surface of a sphere of radius r is given by

$$x^2 + y^2 + z^2 \geq r^2$$

What are the two types of constraints that can be associated with the motion of the particle?

(c) Write down the expression for the Lagrangian of a free particle in cylindrical polar coordinates.

(d) What is the physical significance of the Hamiltonian of a particle?

7. (a) What are cyclic or ignorable coordinates? If a system undergoes translatory motion along a cyclic generalized coordinate q_k, will the Lagrangian of the system be affected? 2

(b) Show that the Poisson bracket of a function with itself is identically zero, i.e., $[u, u] = [v, v] = 0$ where u and v are any two arbitrary functions. 2

Or

Obtain the Lagrangian equation of motion if the Lagrangian has the form $L = -(1 - \dot{q}_j^2)^{1/2}$. Show that the generalized conjugate momentum p_j is conserved. 4

8. Answer any *three* of the following questions : 4×3=12

(a) For a particle subjected to a central force, prove that *(i)* the angular momentum of the particle is a constant of motion, *(ii)* the particle moves in a fixed plane, and *(iii)* the areal velocity of the radius vector remains constant.

(b) The motion of a particle under the influence of a central force is described by $r = a\sin\theta$. Find an expression for the force.

(c) State the d'Alembert's principle. Deduce the Lagrange's equation of motion for a conservative holonomic system using this principle.

(d) The point of suspension of a pendulum moves in the vertically downward direction with constant acceleration a. Find the Lagrangian and hence the equation of motion. What will be its period if the downward acceleration a is the same as that due to gravity?

(e) Show that the Hamiltonian H of a system can be written as

$$H = \sum_j p_j \dot{q}_j - L(q_j, \dot{q}_j, t)$$

where $L(q_j, \dot{q}_j, t)$ is the Lagrangian of the system and p_j are the generalized momenta, q_j are the generalized coordinates and $\dot{q}_j$ are the generalized velocity coordinates.

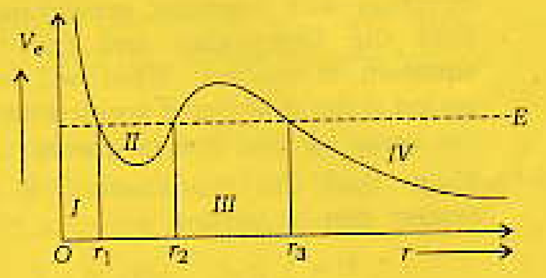
9. Answer any *two* questions : 5×2=10

(a) Assuming attractive inverse square law of force $F(r) = -k/r^2$, where $k > 0$, show that the speed v of the particle in the above field is given by

$$v = \sqrt{\frac{k}{m}\left(\frac{2}{r} \mp \frac{1}{a}\right)}$$

where a is the semi-major axis of the conical path.

(b) Referring to the figure given below, consider an arbitrary potential field caused by a central force. Let us suppose that the total energy E of the particle is represented by the dotted line :



Describe the nature of motion of the particle entering the potential field with energy E in the regions *I*, *II*, *III* and *IV* as shown in the figure. What are turning points of motion?

(c) Using Lagrangian formulation, deduce the equation of motion of a compound pendulum and determine its time period. What is the condition under which the motion of the compound pendulum becomes a simple harmonic motion?

(d) What are the Hamilton's canonical equations of motion? Using Hamilton's canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by $v = -kx$, where k is positive.

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